



# CGC/saturation approach for soft interactions at high energy: Inclusive production



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## ABSTRACT

In this letter we demonstrate that our dipole model is successful in describing inclusive production within the same framework as diffractive physics. We believe that this achievement stems from the fact that our approach incorporates the positive features of the Reggeon approach and CGC/saturation effective theory, for high energy QCD.

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## 1. Introduction

The LHC data on inclusive production [1–3] call for a theoretical understanding of these processes within the framework of QCD. At first sight it would appear, that this process is a typical soft process, which occurs at long distances, where one should use the methods of non-perturbative QCD. Since such methods are only in an embryonic stage, soft processes at high energy remain in the arena of high energy phenomenology, based on the concept of a soft Pomeron. Adopting this approach, inclusive production can be calculated using the technique of Mueller diagrams [4]. It has been demonstrated that soft Pomeron based models provide a reasonable description of the data [5,6]. The advantage of our approach is the feasibility of describing inclusive production on the same footing as diffractive production, and elastic scattering.

On the other hand, in the CGC/saturation approach for inclusive production [7–13], one has a different scenario. In this approach the inclusive production occurs in two stages. The first stage is the production of a mini-jet with the typical transverse momentum  $Q_s$ , where  $Q_s$  is the saturation scale, which is much larger than the soft scale. This process is under full theoretical control.

The second stage is the decay of the mini-jet into hadrons, which has to be treated phenomenologically, using data from the hard processes. Such an approach leads to a good description of the experimental data on inclusive production, both for hadron–hadron, hadron–nucleus and nucleus–nucleus collisions, and observation of regularities in the data, such as geometric scaling [14–18]. The shortcoming of this approach is the fact that it is detached from diffractive physics.

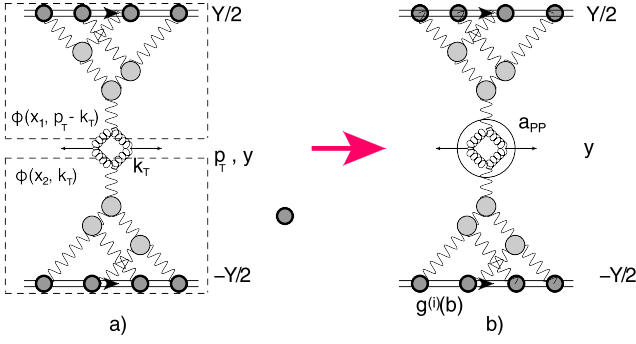
It should be mentioned, that the recently published measurements of the pseudorapidity distributions of charged particles in proton–proton collisions at an energy of 8 TeV provide an additional challenge for model builders, which has not yet been successfully answered [2].

In this letter, we continue (see Refs. [19,20]) to construct a model for high energy soft interactions, which incorporates the advantages of both approaches. This model is based on the Colour Glass Condensate (CGC)/saturation effective theory (see Ref. [21] for the review), and on the perturbative BFKL Pomeron [22]. We assume that the unknown mechanism for the confinement of quarks and gluons in QCD, is not important, and its influence can be reduced to the determination of several parameters related to the CGC/saturation approach, which depend on long distance physics.

The main attributes of the model have been discussed in Refs. [19,20], in this paper we will only include information that we require for the discussion of inclusive production.

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**Fig. 1.** Graphical representation of Eq. (2.1) (see Fig. 1-a). For the sake of simplicity all other indices in  $\phi(x_1, p_T - k_T)$  and  $\phi(x_2, k_T)$  are omitted. The wavy lines denote the BFKL Pomeron, while the helical lines illustrate the gluons. In Fig. 1-b the Mueller diagram for the inclusive production is shown.

## 2. Main formulae

First, we discuss the initial stage of hadron production in the framework of the CGC/saturation approach. For mini-jet production, we use the  $k_T$  factorization formula, that has been proven in Ref. [23] (see also Refs. [24–29] where this proof has been verified).

$$\begin{aligned} \frac{d\sigma}{dy d^2p_T} &= \frac{2\pi\tilde{\alpha}_S}{C_F} \frac{1}{p_T^2} \int d^2k_T \phi_G^{h_1}(x_1; \vec{k}_T) \phi_G^{h_2}(x_2; \vec{p}_T - \vec{k}_T) \\ &= \frac{2C_F}{\alpha_s(2\pi)^4} \frac{1}{p_T^2} \int d^2r e^{i\vec{p}_T \cdot \vec{r}} \nabla_r^2 N_G^{h_1}(y_1; r; t=0) \\ &\quad \nabla_r^2 N_G^{h_2}(Y - y_1; r; t=0) \end{aligned} \quad (2.1)$$

where  $\phi_G^{h_i}$  denotes the probability to find a gluon that carries the fraction  $x_i$  of energy with  $k_\perp$  transverse momentum, and  $\tilde{\alpha}_S = \alpha_s N_c / \pi$ , with the number of colours equal to  $N_c$ .  $\frac{1}{2}Y + y = \ln(1/x_1)$  and  $\frac{1}{2}Y - y = \ln(1/x_2)$ .  $y$  is the rapidity of the emitted gluon in the c.m., while  $\frac{1}{2}Y$  and  $-\frac{1}{2}Y$  denote the rapidities of the colliding hadrons (see Fig. 1). As can be seen from Eq. (2.1) the rapidities  $\frac{1}{2}Y - y$  and  $\frac{1}{2}Y + y$  determine the energy of the dipole scattering.  $N_G^{h_i}(y_i, r, t=0)$  denotes the forward scattering (at  $t=0$ ) amplitude of the dipole with size  $r$ .  $\phi_G^{h_i}$  and  $N_G^{h_i}(y_i, r, t=0)$  are the solutions of the Balitsky–Kovchegov (BK) [10,12] non-linear evolution equation in the momentum and coordinate representations, and can be viewed as the sum of ‘fan’ diagrams of the BFKL Pomeron interactions, shown in Fig. 1-a. Using this fact we can re-write Eq. (2.1) as the Mueller diagram of Fig. 1-b absorbing the integration over  $k_T$  (or over the dipole size  $r$ ), into the new phenomenological vertex  $a_{PP}$ .

In our model we use the simple formula which is a good approximation to the numerical solution of the BK equation, see Ref. [30]: viz.

$$N^{BK}(G_{\mathbb{P}}(z)) = a(1 - \exp(-G_{\mathbb{P}}(z))) + (1-a) \frac{G_{\mathbb{P}}(z)}{1 + G_{\mathbb{P}}(z)}, \quad (2.2)$$

with  $a = 0.65$  and  $G_{\mathbb{P}}(z = \ln(r^2 Q_s^2(y, b))) = \phi_0(r^2 Q_s^2(y, b))^{1-\gamma_{cr}}$  where we have used two inputs:  $r = R$  and  $Q_s^2 = (1/(m^2 R^2)) S(mb) \exp(\lambda y)$ . For the values of  $1 - \gamma_{cr}$  and  $\lambda$ , we have estimates in the leading order of perturbative QCD:  $1 - \gamma_{cr} = 0.63$  and  $\lambda = 4.88\tilde{\alpha}_S$ . The value of  $\lambda$  is a fitting parameter, which effectively includes the higher order QCD corrections. In this paper we use the value  $\lambda = 0.38$  which we determined in Ref. [20].

The parameter  $m$  and the function  $S(mb)$  originate from non-perturbative QCD contributions, and are conjectured to be of the following form:

$$S(mb) = \frac{m^2}{\pi^2} e^{-mb} \quad \text{where} \quad \int d^2b S(b) = 1 \quad (2.3)$$

$\phi_0$  can be calculated using the initial conditions, of the BFKL equation.

However, we do not know these conditions, and so we consider  $\phi_0$  as an additional phenomenological parameter. The values of these parameters are taken from Ref. [20]:  $m = 5.25$  GeV and  $\phi_0 = 0.0019$ . All these parameters describe the CGC /saturation structure of the BFKL Pomeron and their interactions. We introduce phenomenological parameters to describe the structure of the hadron. We choose the two channel model for such a structure, and describe the vertex of the BFKL Pomeron interaction with a hadron state  $i$  in the following form:

$$\begin{aligned} g_i(m_i, b) &= g_i S_{\mathbb{P}}(m_i, b) \quad \text{where} \\ S_{\mathbb{P}}(m_i, b) &= \frac{1}{4\pi} m_i^2 b K_1(m_i b) \end{aligned} \quad (2.4)$$

The parameters that we use in this paper, have been extracted from fitting the elastic and diffractive data in Ref. [20], and their values are:

$$\begin{aligned} g^{(1)} &= 110.2 \text{ GeV}^{-1}; \quad m_1 = 0.92 \text{ GeV}; \\ g^{(2)} &= 11.2 \text{ GeV}^{-1}; \quad m_2 = 1.9 \text{ GeV}; \end{aligned} \quad (2.5)$$

Finally, Eq. (2.1) can be re-written as a Mueller diagram of Fig. 1-b, and the inclusive cross section is given by

$$\begin{aligned} \frac{d\sigma}{dy} &= \int d^2p_T \frac{d\sigma}{dy d^2p_T} \\ &= a_{PP} \ln(W/W_0) \left\{ \alpha^4 \ln^{(1)}\left(\frac{1}{2}Y + y\right) \ln^{(1)}\left(\frac{1}{2}Y - y\right) \right. \\ &\quad + \alpha^2 \beta^2 \ln^{(1)}\left(\frac{1}{2}Y + y\right) \ln^{(2)}\left(\frac{1}{2}Y - y\right) \\ &\quad + \ln^{(2)}\left(\frac{1}{2}Y + y\right) \ln^{(1)}\left(\frac{1}{2}Y - y\right) \\ &\quad \left. + \beta^4 \ln^{(2)}\left(\frac{1}{2}Y + y\right) \ln^{(2)}\left(\frac{1}{2}Y - y\right) \right\} \end{aligned} \quad (2.6)$$

where  $\alpha$  and  $\beta$  describe the structure of the diffractive scattering in the two channel model, where the observed physical hadronic and diffractive states are written in the form

$$\begin{aligned} \psi_h &= \alpha \Psi_1 + \beta \Psi_2; \quad \psi_D = -\beta \Psi_1 + \alpha \Psi_2; \quad \text{where} \\ \alpha^2 + \beta^2 &= 1; \end{aligned} \quad (2.7)$$

$\ln^{(i)}$  is given by

$$\ln^{(i)}(y) = \int d^2b N^{BK}(g^{(i)} S(m_i, b) \tilde{G}_{\mathbb{P}}(y)) \quad (2.8)$$

where  $\tilde{G}_{\mathbb{P}}(y) = \phi_0 \exp(\lambda(1 - \gamma_{cr})y)$  and  $N^{BK}$  is defined in Eq. (2.2). Regarding the factor in front of Eq. (2.6) i.e.  $\ln(W/W_0)$ , where  $W = \sqrt{s}$  is the energy of collision in c.m.f., and  $W_0$  is the value of energy from which we can start our approach. One can see that Eq. (2.1) is divergent in the region of small  $p_T < Q_s$ . Indeed, in this region  $\phi$ 's in Eq. (2.1) do not depend on  $p_T$ , since  $k_T \approx Q_s > p_T$ , and the integration over  $p_T$  leads to  $\ln(Q_s^2/m_{soft}^2)$ ,

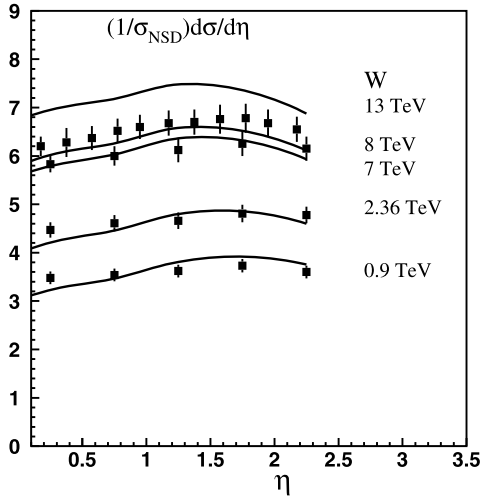


Fig. 2-a

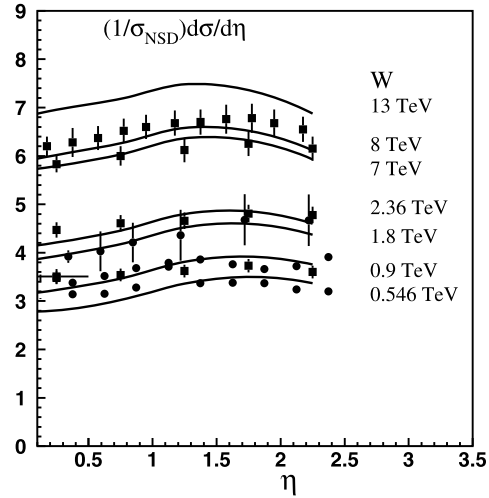


Fig. 2-b

**Fig. 2.** The single inclusive density  $((1/\sigma_{\text{NSD}})d\sigma/d\eta)$  versus energy. The data were taken from Refs. [1–3] and from Ref. [31]. The description of the CMS data is plotted in Fig. 2-a, while Fig. 2-b presents the comparison with all inclusive spectra with  $W \geq 0.546$  TeV.

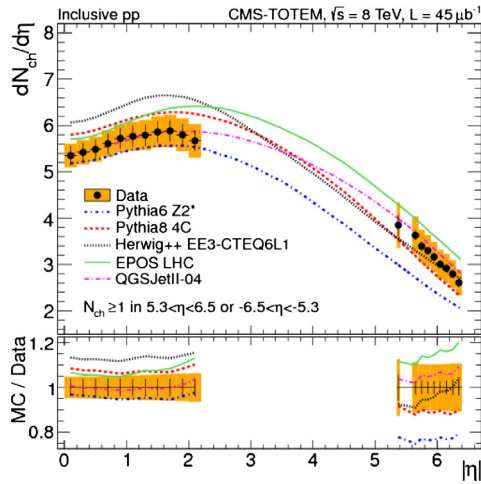


Fig. 3-a

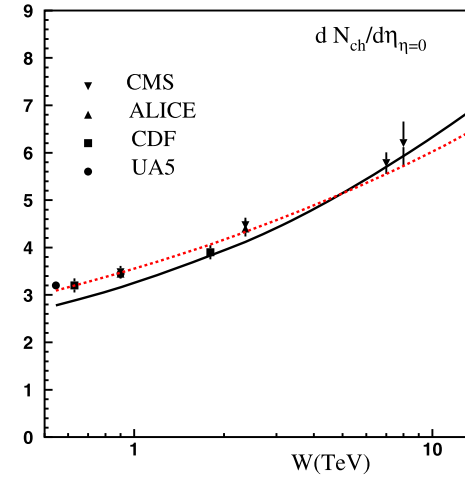


Fig. 3-b

**Fig. 3.** The comparison of the inclusive production at  $W = 8$  TeV with Monte Carlo models is shown in Fig. 3-a. The figure is taken from Ref. [2]. In Fig. 3-b we show  $dN_{\text{ch}}/d\eta$  at  $\eta = 0$  versus energy  $W$ . Our estimates are shown by the solid line. The dotted line corresponds to fit:  $0.725(W/W_0)^{0.23}$  with  $W_0 = 1$  GeV (see [2]). The data are taken from Refs. [2,1,32,33].

where  $m_{\text{soft}}$  is the non-perturbative scale that includes the confinement of quarks and gluons ( $m_{\text{soft}} \sim \Lambda_{\text{QCD}}$ ).

To convert the rapidity distribution (which we calculate theoretically), to pseudorapidity ( $\eta$ ) one, we need to know the mass of mini-jet ( $m_{\text{jet}}$ ). The simple formulae are well known (see Ref. [14] for example):

$$y(\eta, p_T) = \frac{1}{2} \ln \left\{ \frac{\sqrt{\frac{m_{\text{jet}}^2 + p_T^2}{p_T^2} + \sinh^2 \eta} + \sinh \eta}{\sqrt{\frac{m_{\text{jet}}^2 + p_T^2}{p_T^2} + \sinh^2 \eta} - \sinh \eta} \right\} \quad (2.9)$$

with the Jacobian

$$h(\eta, p_T) = \frac{\cosh \eta}{\sqrt{\frac{m_{\text{jet}}^2 + p_T^2}{p_T^2} + \sinh^2 \eta}} \quad (2.10)$$

The mass of mini-jet is given by  $m_{\text{jet}}^2 = 2m_{\text{soft}}p_T$  (see Ref. [14]). Since the typical transverse momentum is equal to the saturation scale, we have

$$\frac{m_{\text{jet}}^2}{p_T^2} = \frac{2m_{\text{soft}}}{Q_s(W)} = r_0^2 \left( \frac{W}{W_0} \right)^{-\frac{1}{2}\lambda} \quad (2.11)$$

where  $r_0^2$  and  $a_{\text{pp}}$  are phenomenological parameters that are determined by fitting to the experimental data.

Finally,

$$\frac{d\sigma}{d\eta} = h(\eta, Q_s) \frac{d\sigma}{dy}(y(\eta, Q_s)) \quad (2.12)$$

### 3. Comparison with the experimental data

In Fig. 2 we plot our predictions compared to the experimental data. As we have mentioned all other parameters have been extracted from the diffractive and elastic data in Ref. [20]. The only free parameters are  $a_{\text{pp}}$  and  $r_0^2$ . We wish to emphasize that both

of these parameters have not been fixed from the cross section data in our previous papers [19,20] and they describe:  $a_{\text{pp}}$  the result of integration over  $k_T$  (or  $r$ ) in Eq. (2.1); and  $r_0$  the gluon jet decay into hadrons. In principle, they could be calculated using more detailed input from high energy QCD for the dipole scattering amplitude and jet decay, but in the framework of this paper we extract their values from the fit to the experimental data.

The curves in Fig. 2 are calculated for  $a_{\text{pp}} = 0.21$  and  $r_0^2 = 8$ .

From Eq. (2.6), we note that the inclusive cross section is sensitive to the contribution of the black component. As we discussed in Ref. [20], qualitatively, we have in our two channel model two different components: one which is transparent, even at ultra high energy (e.g. at  $W = 57$  TeV) while the second component, starts being black at rather low energy (say at  $W = 0.9$  TeV). Hence, our good description of the experimental data, checks that the value of this component is consistent with the inclusive data.

One can see that our model describes the value of the inclusive densities  $\rho = (1/\sigma_{\text{NSD}})d\sigma/d\eta$  and their dependence on energy and rapidity, rather well. It should be stressed that the values for  $\sigma_{\text{NSD}}$  were calculated in our model. Regarding the new data at  $W = 8$  TeV (Figs. 2 and 3), the comparison shows that the result of our approach, is slightly below the experimental central values, while the numerous Monte Carlo simulations overshoot the data in the central region (see Fig. 3-a).

Note, we have only dealt with data in the central region, since we do not take into account parton correlations due to energy conservation. These are important in the fragmentation region, but difficult to incorporate in our present framework.

In Fig. 3-b we show the energy dependence of  $dN_{\text{ch}}/d\eta$  at  $\eta = 0$ . In the CGC/saturation approach  $dN_{\text{ch}}/d\eta|_{\eta=0} \propto W^\lambda$ , where  $\lambda$  corresponds to the energy dependence of the saturation scale. In our model the energy dependence is more complicated and can be approximate as  $W^{0.29}$ . Note, that the power 0.29 is much less than the value of  $\lambda$  ( $\lambda = 0.38$ ). It is worthwhile mentioning that we can also estimate  $(1/\sigma_{\text{in}})d\sigma/d\eta \equiv dN_{\text{inel}}/d\eta$  where  $\sigma_{\text{in}} = \sigma_{\text{tot}} - \sigma_{\text{el}}$ . It turns out that the energy dependence of this multiplicity is milder and can be described as  $W^{0.27}$ . Qualitatively this estimate is in agreement with the ALICE data [1,34], which shows that the behaviour of the inelastic multiplicity is proportional  $W^{0.2}$  instead of  $W^{0.23}$ , as we saw for the multiplicity in NSD events in Fig. 3-b.

Concluding this discussion, we see that we obtain a good description of the data, but our estimates are below the data at small values of rapidity  $\eta$ . We believe, that this is a reflection of our simplified relation between  $y$  and  $\eta$ .

#### 4. Conclusions

In this letter we demonstrate that our model for the soft (long distance) interaction which is based on CGC/saturation approach, is able to describe inclusive production. In other words, we give the example that our model can describe both the diffractive (elastic) physics at high energy, and the typical production process, in the majority of other approaches these are treated in different ways. We believe that the reason for the success of our approach has its roots in the fact, that our procedure incorporates the advantages of high energy phenomenology based on the soft Pomeron interactions, and of the CGC/saturation effective theory that includes the description of multi-particle production in perturbative QCD.

This letter is a natural step in our search for a model based on QCD, that will be able to describe the typical properties of high energy interactions, and includes diffractive production and multi-particle generation processes on the same footing. It is also a next step in our attempts to build this description without addressing Monte Carlo simulation methods.

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